The added-mass effect that an encompassing fluid exerts on a moving solid body was intensively investigated in the 19-th century in the context of the motion of pendulums. The first written account of this effect appears to be the paper by Stokes, read before the Cambridge Philosophical Society in 1850 [6], although this paper mentions earlier work by Bessel from 1828. A side note of historical interest is that Stokes’ analysis in [6] is based on equations that he had derived five years earlier and that have come to bear his name together with that of Navier.

Evidently, pendulums have dwindled as an object of active scientific investigation. The added mass effect has however recently regained interest on account of its influence on the convergence behavior of iterative solution methods for fluid-structure-interaction (FSI) problems. The ratio of the added mass to the structural mass is critical to the convergence-and-stability properties of partitioned iterative solution methods for FSI. In the basic iterative method for FSI problems, the fluid and solid subproblems are solved alternatingly subject to complementary partitions of the interface conditions. The convergence rate of this so-called subiteration process coincides with the fluid-structure mass ratio. Therefore, subiteration is unstable if the fluid-structure mass ratio exceeds one while, conversely, it provides very effective convergence for FSI problems in which the added mass of the fluid is much smaller than the structural mass.

The added mass of incompressible flows and its effect on the convergence behavior of partitioned solution methods has been investigated in [1, 3, 5]. In [1], we have in addition established the added-mass effect of compressible flows.

$10 \log$ of the error $e_n$ in the subiteration process versus the iteration counter $n$ for fluid-structure mass ratios (from bottom to top) $\sigma = 2^{-2}, 2^{-1}, \ldots, 2^{4}$ (see [2]).
Our analysis conveys that, essentially, the added mass of a compressible flow is proportional to the duration of the time interval under consideration (i.e., the time step in the numerical time-integration procedure), whereas the added mass of an incompressible flow approaches a constant as the time interval vanishes. Consequently, regardless of the density of the fluid and the mass of the structure, for compressible flows the subiteration process is stable and convergent for sufficiently small time steps. For incompressible flows, this is not the case, and the subiteration method can remain unstable in the limit of vanishing time-step size. This implies, in particular, that loosely-coupled (or staggered) time-integration schemes, in which only a single iteration per time step is performed [4], are in general suitable for compressible-fluid-structure-interaction problems, but not for incompressible-fluid-structure-interaction problems. In [2] we have conducted a refined analysis of the subiteration operator for compressible- and incompressible-flow models, to serve as a basis for the analyses of several partitioned solution methods which use subiteration as a component. We established that for compressible flows, subiteration corresponds to a Volterra operator, which is quasi-nilpotent and nonnormal. This implies that for compressible flows and large time steps, the subiteration method can yield non-monotonous convergence.

To illustrate the convergence behavior of subiteration for compressible flows, Figure 1 plots the error $\varepsilon_n$ versus the iteration counter $n$ for mass ratios $\rho C T / \mu =: \sigma = 2^{-2}, 2^{-1}, \ldots, 2^4$ with $\rho$ as fluid density, $C$ as speed of sound, $T$ as time step and $\mu$ as structure density. At large time steps, transient divergence occurs before asymptotic convergence sets in, as a result of non-normality. The convergence behavior improves as the time step and, accordingly, the added-mass decreases.

**References**