In simulations of compressible flow with a standard finite-volume method, small turbulent flow structures can transfer internal energy to kinetic energy through convective transport. This energy transfer violates a conservation property of convective transport, and may eventually cause hopeless numerical instability of a simulation. Therefore, finite-volume methods often need to eliminate small flow structures through artificial dissipation to attain numerical stability. However, although artificial dissipation can successfully stabilize a finite-volume method, it can also have undesirable side-effects such as numerical delay of the transition to turbulence or attenuation of acoustic waves.

The needed level of artificial dissipation can be reduced by enforcing conservation of kinetic and internal energy by convective transport at the discrete level. We found that this can be done straightforwardly if the state of a compressible fluid is expressed in the square root variables $\sqrt{\rho}$, $\sqrt{\rho} \frac{u}{\sqrt{2}}$, and $\sqrt{\rho} e$. In these variables the conservation of mass, momentum, kinetic energy, internal energy and total energy by convective transport can all be explained from a mathematical skew-symmetry. If this skew-symmetry is preserved by a simulation method, then discrete convective transport does not only conserve mass, momentum, and total energy as in a standard finite-volume method, but also kinetic and internal energy separately.

This contour plot of the axial vorticity above the delta wing shows the primary vortex and the upper surface boundary layer (blue) which is sucked into the primary vortex. Although some light spurious wiggles can be observed, these do not cause numerical instability because the simulation method preserves symmetries.
A skew-symmetry-preserving simulation method needs little artificial dissipation for stability. This is demonstrated in simulations of the transitional compressible flow over a delta wing. The flow over a delta wing is dominated by two primary vortices, which are formed by roll-up of the shear layer that emanates at the leading edge of the wing. The simulation method is fourth-order accurate, the computational grid stretches and bends considerably, and the primary vortices feature many small flow structures. However, because the skew-symmetry of convective transport is preserved at the discrete level, the simulation is stable without artificial dissipation close to the delta wing. Because the simulation method understands that convective transport conserves kinetic energy, artificial dissipation is an option and not a necessary condition for numerical stability.

REFERENCES

The transitional flow over a delta wing at Re = 50,000 visualized as an iso-surface of the q-criterion colored by the axial vorticity.